Technical Notes

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Quadrature Formulas for Chordwise Integrals of Lifting Surface Theories

Shigenori Ando* and Akio Ichikawa†
Nagoya University, Nagoya, Japan

Introduction

IT is well known that all of the following integrals appear in subsonic unsteady lifting airfoil theories

$$I^{(1)} = \int_{-1}^{1} \sqrt{\frac{1-\xi}{1+\xi}} f(\xi)\,d\xi \tag{1}$$

$$I^{(2)}(x) = \int_{-1}^{1} \sqrt{\frac{1-\xi}{1+\xi}} \frac{f(\xi)}{x-\xi}\,d\xi \tag{2}$$

$$I^{(3)}(x) = \int_{-1}^{1} \sqrt{\frac{1-\xi}{1+\xi}} f(\xi)\ln|x-\xi|\,d\xi \tag{3}$$

In this Note we discuss applications of these integrals to cases where the doublet-lattice method is adopted. Obviously, the layouts of both the loading and upwash points for Eqs. (1-3) must be selected uniformly. Our main interest lies in $I^{(3)}$, but $I^{(1)}$ and $I^{(2)}$ will also be discussed for comparison.

Integral $I^{(1)}$

First we know the familiar quadrature formula,[1,2]

$$I^{(1)} = \int_{-1}^{1} \sqrt{\frac{1-\xi}{1+\xi}} f(\xi)\,d\xi = \sum_{j=1}^{N} A_j^{(1)} f(\xi_j) + O(f^{(2N)})$$

where

$$A_j^{(1)} = \frac{2\pi}{2N+1}(1-\xi_j) \tag{4}$$

$$\xi_j = -\cos\frac{2j-1}{2N+1}\pi, \qquad j=1,2,\ldots N$$

The ξ_j yield the loading points, while the corresponding upwash points are

$$x_p = -\cos\frac{2p\pi}{2N+1}, \qquad p=1,2,\ldots N \tag{5}$$

Integral $I^{(2)}(x)$

This integral is treated in several ways:

1) Hsu's method. Rewriting $f(\xi)=[f(\xi)-f(x)]+f(x)$, Hsu[1] derives

$$I^{(2)}(x) = \pi f(x) + \int_{-1}^{1} \sqrt{\frac{1-\xi}{1+\xi}} \frac{f(\xi)-f(x)}{x-\xi}\,d\xi \tag{6}$$

the second term of which is treated by use of $I^{(1)}$. Thus

$$I^{(2)}(x_p) = f(x_p)\left[\pi - \sum_{j=1}^{N} \frac{A_j^{(1)}}{x_p-\xi_j}\right] + \sum_{j=1}^{N} \frac{A_j^{(1)} f(\xi_j)}{x_p-\xi_j} \tag{7}$$

which may be reduced to

$$I^{(2)}(x_p) = \sum_{j=1}^{N} \frac{A_j^{(1)} f(\xi_j)}{x_p-\xi_j} + O(f^{(2N+1)}) \tag{8}$$

as shown later. It seems that the change from Eq. (7) to Eq. (8) was not noticed by previous writers, with the possible exception of Stark.[3]

2) Our derivation. A regular function may be expressed using the interpolation functions

$$f(\xi) = \sum_{j=1}^{N} g_j(\xi) f(\xi_j) + O(f^{(N)}) \tag{9}$$

where the $g_j(\xi)$ selected here is the interpolation function, a polynomial of degree $N-1$ concerning the weight function $\sqrt{(1-\xi)/(1+\xi)}$. Using

$$x = -\cos\theta \quad \text{and} \quad \xi = -\cos\varphi \tag{10}$$

and simply writing $g_j(-\cos\varphi)$ as $g_j(\varphi)$, etc., we have[2]

$$g_j(\varphi) = \frac{2(-1)^{j+1}}{2N+1} \frac{\sin\varphi_j \cos\varphi_j/2}{\cos\varphi - \cos\varphi_j} \frac{\cos(N+\frac{1}{2})\varphi}{\cos\varphi/2} \tag{11}$$

Then

$$I^{(2)}(x) = \sum_{j=1}^{N} f(\varphi_j) \int_0^{\pi} \frac{g_j(\varphi)(1+\cos\varphi)}{\cos\varphi - \cos\theta}\,d\varphi \tag{12}$$

Assuming $\varphi_j \neq \theta$ and using some elementary integrations and manipulations, we have

$$I^{(2)}(x) = \sum_{j=1}^{N} A_j^{(2)}(\theta) f(\varphi_j) + O(f^{(N)}) \tag{13}$$

where

$$A_j^{(2)}(\theta) = \frac{2\pi}{2N+1} \frac{1-\xi_j}{x-\xi_j} \left[1 + (-1)^j \frac{\sin\varphi_j/2}{\sin\theta/2} \sin(N+\tfrac{1}{2})\theta\right] \tag{14}$$

The value of θ is still not specified, except for $\theta \neq \varphi_j$. If θ as in Eq. (5) is selected, $A_j^{(2)}$ reduces to $A_j^{(1)}/(x_p-\xi_j)$ and Eq. (13) to Eq. (8). It should be noted that Eq. (8) is Stark's formula[3] with the special weight function

$$W(\xi) = \sqrt{(1-\xi)/(1+\xi)}$$

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*Professor, Department of Aeronautical Engineering. Member AIAA.

†Research Associate, Department of Aeronautical Engineering. Member AIAA.

3) Lan's method. Lan[4] derived the following expression

$$I^{(2)}(x_i)=\frac{\pi}{N}\sum_{k=1}^{N}\frac{1-\xi_k}{x_i-\xi_k}f_k$$

$$\xi_k=-\cos\frac{(2k-1)}{2N}\pi,\qquad x_i=-\cos\frac{i\pi}{N}$$

$$k=1,2,\ldots N,\qquad i=1,2,\ldots N \tag{15}$$

Lan used the trapezoidal rule for trigonometric variables in place of interpolation functions, as well as Chebychev's polynomial of the first kind. Note that Eq. (15) leads to a scheme different from that in Eqs. (4) and (8) for the layout of loading and upwash points.

Integral $I^{(3)}(x)$

$I^{(3)}(x)$ containing the logarithmic kernel is described by Fromme and Golberg[5] as a pressure mode scheme. But we want it in doublet-lattice scheme. Use of Eq. (10) and the formula

$$\ln|\cos\theta-\cos\varphi|=-2\sum_{n=1}^{\infty}\frac{1}{n}\cos n\theta\cos n\varphi-\ln 2\quad\text{for }\theta\neq\varphi \tag{16}$$

yields

$$I^{(3)}(x)=\sum_{n=0}^{\infty}F_n(\theta)\int_0^{\pi}f(\varphi)\cos n\varphi\,\mathrm{d}\varphi \tag{17}$$

where

$$-F_n(\theta)=\ln 2+\cos\theta,\qquad n=0$$

$$=\ln 2+2\cos\theta+\tfrac{1}{2}\cos 2\theta,\qquad n=1$$

$$=\frac{\cos(n-1)\theta}{n-1}+\frac{2}{n}\cos n\theta+\frac{\cos(n+1)\theta}{n+1},\quad n\geq 2 \tag{18}$$

Substitution of Eq. (9) into the integral in Eq. (17) gives

$$\int_0^{\pi}f(\varphi)\cos n\varphi\,\mathrm{d}\varphi=\frac{2\pi}{2N+1}\sum_{j=1}^{N}f(\varphi_j)$$

$$\times\left[\cos n\varphi_j+(-1)^{N+j+n}\sin\frac{\varphi_j}{2}\right] \tag{19}$$

Thus we have

$$I^{(3)}(x)=\sum_{j=1}^{N}A_j^{(3)}(x)f(\xi_j)\quad\text{for }x\neq\xi_j \tag{20a}$$

where

$$A_j^{(3)}(x)=\frac{2\pi}{2N+1}\sum_{n=0}^{N}\left[\cos n\varphi_j+(-1)^{N+j+n}\sin\frac{\varphi_j}{2}\right]F_n(\theta) \tag{20b}$$

In our work on calculations of subsonic nonsteady airfoil loading with the doublet-lattice scheme, the quadrature of Eqs. (20) with $x=x_p$ drastically improves the convergence of solutions. Truncation of the n summation by N is seen to be quite satisfactory in several applications.[6]

References

[1] Hsu, P. T., "Some Recent Developments in the Flutter Analysis of Low-Aspect-Ratio Wings," Paper presented at Institute of the Aeronautical Sciences, National Specialists Meeting, 1958.

[2] Davies, D. E., "Calculation of Unsteady Generalized Air Forces on a Thin Wing Oscillating Harmonically in Subsonic Flow," British Aeronautical Research Council, R&M 3409, 1965.

[3] Stark, V.J.E., "A Generalized Quadrature Formula for Cauchy Integrals," *AIAA Journal*, Vol. 9, Sept. 1971, pp. 1854-1855.

[4] Lan, C. E., "A Quasi-vortex-Lattice Method in Thin Wing Theory," *Journal of Aircraft*, Vol. 11, Sept. 1974, pp. 518-527.

[5] Fromme, J. and Golberg, M., "Unsteady Two-Dimensional Airloads Acting on Oscillating Thin Airfoils in Subsonic Ventilated Wind Tunnels," NASA CR 2967, 1978.

[6] Ando, S. and Ichikawa, A., "The Use of an Error Index to Improve Numerical Solutions for Unsteady Lifting Airfoils," *AIAA Journal*, Vol. 21, Jan. 1983, pp. 47-54.

Freestream Turbulence and Transonic Flow over a "Bump" Model

S. Raghunathan* and R.J.W. McAdam†

The Queen's University of Belfast, Northern Ireland

Nomenclature

C_p = nondimensional pressure coefficient
ℓ_s = separated flow length
M_{pk} = peak Mach number on the model
M_∞ = freestream Mach number measured two chords upstream of model leading edge
R_θ = Reynolds number based on θ
R_δ = Reynolds number based on δ
Tu_∞ = freestream turbulence intensity 6 mm upstream of the shock location, $\tilde{u}/U_\infty$
X = distance measured from leading edge of the model
X_s = shock position measured from the leading edge of the model
X_{s0} = shock position at zero turbulence intensity
$\tilde{u}$ = root mean square velocity fluctuation
U_∞ = freestream velocity 6 mm upstream of the shock location
δ = boundary-layer thickness 6 mm upstream of the shock location
δ^* = boundary-layer displacement thickness 6 mm upstream of the shock location
θ = boundary-layer momentum thickness 6 mm upstream of the shock location

Introduction

IN transonic flow, predictions of free-flight conditions from wind tunnel tests need a close simulation of Reynolds number. However, Reynolds number simulation is not adequate, since results from wind tunnel tests are influenced by effects of tunnel environment, such as noise and turbulence levels.[1] A recent paper by the authors[2] showed a strong influence of turbulence on attached turbulent boundary layers at zero pressure gradient in the Mach number range of 0-0.8 and boundary-layer momentum thickness Reynolds number range of 3000-10^4. This Note presents the general characteristics of transonic flow over a "bump" model at various freestream turbulence levels.

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*Lecturer, Department of Aeronautical Engineering. Member AIAA.

†Research Student, Department of Aeronautical Engineering.